Turbulent free convection above a heated plate inclined at a small angle to the horizontal

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An investigation has been made of the structure of the motion above a heated plate inclined at a small angle (about 10°) to the horizontal. The turbulence is considered in terms of the similarities to and differences from the motion above an exactly horizontal surface. One effect of inclination is, of course, that there is also a mean motion.

Accurate data on the mean temperature field and the intensity of the temperature fluctuations have been obtained with platinum resistance thermometers, the signals being processed electronically. More approximate information on the velocity field has been obtained with quartz fibre anemometers. These results have been supplemented qualitatively by simultaneous observations of the temperature and velocity fluctuations and also by smoke experiments.

The principal features of the flow inferred from these observations are as follows. The heat transfer and the mean temperature field are not much altered by the inclination, though small, not very systematic, variations may result from the complexities of the velocity field. This supports the view that the mean temperature field is largely governed by the large-scale motions. The temperature fluctuations show a systematic variation with distance from the lower edge and resemble those above a horizontal plate when this distance is large. The large-scale motions of the turbulence start close to the lower edge, but the smaller eddies do not attain full intensity until the air has moved some distance up the plate. The mean velocity receives a sizable contribution from a 'through-flow' between the side-walls. Superimposed on this are developments that show that the momentum transfer processes are complex and certainly not capable of representation by any simple theory such as an eddy viscosity. On the lower part of the plate there is a surprisingly large acceleration, but further up the mixing action of the small eddies has a decelerating effect.

1. Preliminary remarks

There has been set up in the Cavendish Laboratory, Cambridge (where the work described here was done) a heated flat plate, that can be set to any inclination, for free-convection studies. This paper describes one particular set of experiments that was carried out with this apparatus, an investigation of the flow above the plate when its angle with the horizontal was around 10°. This was selected at an early stage in an (uncompleted) investigation of the changes that occur as the inclination is varied through the full 180° range, because of the information available for the limiting case of an exactly horizontal surface. In fact, when the experiment was started, it was hoped that the way in which the flow changed when the plate was inclined a little might help towards settling a controversy about the horizontal plate flow. This hope was not fulfilled; complications invalidated the assumptions on which the argument was based. However, out of a rather involved overall picture, there emerge some points that seem of basic interest. There are also several open questions. The reason that the work is being reported in its present state is partly just that, for personal reasons, it is not at the moment being continued. Also, however, any further investigation probably ought to be preceded by an effort to eliminate those features, revealed by my work, that are due only to the end and side conditions of the particular configuration (though it is not obvious how this should be done). This paper, therefore, gives probably as complete a description of the flow with one configuration as is worth the effort of obtaining.

There has been very little previous work with this arrangement. Schmidt's (1932) fine collection of schlieren photographs of free convective flows includes the case of a plate at a small angle to the horizontal; but the plate was small so the flow pictured corresponds to that on only the lowest part of my plate. Tautz (1941) experimented with plates inclined at various angles, but measured only the combined heat transfer from the upper and lower surfaces.†

2. Notation

Cartesian co-ordinates are used with x up the line of greatest slope ($x = 0$ at the lower edge), y the distance measured normally from the plate,‡ and z parallel to the lower edge. u, v and w are the corresponding components of velocity. Other symbols used are:

g	accleration due to gravity
h	deflexion of the free end of a fibre anemometer
k	thermal conductivity of air
n	exponent in equation (1) (§5)
u_0	velocity scale introduced in §§5 and 6
$u_{\max}$	the maximum velocity with respect to y (at fixed x)
y_0	length scale introduced in §§5 and 6
$y_{\max}$	the distance from the plate of the position of the velocity maximum
C	constant in heat-transfer equations (§§5 and 8)
C_p	specific heat of air at constant pressure
Gr	Grashof number (see §6)
H	heat transfer per unit area from plate
Pr	Prandtl number, ν/κ
Q	$H/\rho C_p T$
T	air temperature
T_0	ambient temperature

† I know of this paper only through the summary given by Kraus (1955, pp. 120, 122, 135), so it may contain other relevant information.

‡ y is retained as the distance from the plate even when a horizontal plate is being considered; the use of the conventional z might be confusing.

T_1 plate temperature
T_a temperature at top of side-walls (one possible value for T_0)
T_r room temperature (another possible value for T_0)
α angle of plate to vertical, taken positive for flow below†
δ boundary-layer thickness (defined more specifically when necessary)
θ_0 scale of $(T-T_0)/T_0$ introduced in §§5 and 6
κ thermometric conductivity ($k/\rho C_p$)
ν kinematic viscosity
ρ density of air.

The superscripts $^-$ and $'$ apply to a quantity that fluctuates in turbulence and indicate respectively the time mean and the instantaneous deviation from the mean. A symbol without superscript denotes the instantaneous value. $\tilde{u}$ denotes the 'measured' velocity, to be introduced in §4(*d*).

3. The heated plate

Since other experiments with the same apparatus are described elsewhere (Tritton 1963*b*), it will be convenient to describe the apparatus in a rather more general way than is directly applicable to the topic of this paper.

The fluid used is air. The apparatus is constructed in a sandwich arrangement with surfaces suitable for convection experiments on either side (allowing an immediate comparison of the flow above and below). These surfaces are plates of 'Noral' aluminium alloy. The length (along the line of greatest slope) is 183 cm and the width 30 cm. There are a few nuts and other obstacles (connected with the mounting of the plate and traversing gear) along the edges of the two surfaces, but the centre 22 cm on top and the centre 24 cm below are quite clear.

There are (transparent) side-walls both above and below. Without these the flow above the plate would be dominated by a buoyant plume rising off the surface with inflow from either side; similarly, there would be an outward flow on the underside. On the other hand, we shall see that the use of side-walls is not entirely successful (particularly for the flow described in the present paper) in making the flow approximate to the ideal on an infinitely wide plate. But for reasons of economy and ease of construction a wider plate would be better, though, in view of flow behaviour to be described in §11, it is doubtful if this alone would overcome the difficulties.

The top and bottom edges of the plates are of course open, but an attempt has been made to choose the end conditions that bring the flow closest to the ideal on a plate of semi-infinite length in the flow direction; i.e. to make the distance from the lower edge, not that from the upper, the governing length-scale. It is mainly in the flow above the plate at small angles to the horizontal that this does not come about automatically, so the point will be taken up again in §7, where descriptions of different arrangements will be given together with their effect on

† This definition of α is rather inconvenient for this paper, but has been used so as to have a single notation for all the work on inclined plates. In general, it is useful to distinguish flow below and flow above by the sign of α, and further to make the signs of $\sin\alpha$ and the Richardson number the same.

the air flow. It is also, of course, a function of any arrangement to prevent the flow on the other side of the plate from interfering.

The working surfaces are nominally smooth to 0·005 cm.; odd scratches may make them a little worse than that. There is a join at the centre (lengthways) of each surface; although this was originally made smooth, repeated heating and cooling of the plate has deformed it and in some of the work there has been a 'step' of up to 0·02 cm. The surfaces are kept polished as this is the best way of ensuring that conditions are repeatable.

The heating is electrical; the elements at the centre of the 'sandwich' are 32 s.w.g. nichrome wires running laterally at a spacing of about $1\frac{1}{2}$ cm and are separated from the aluminium plates on either side by layers of asbestos about $1\frac{1}{4}$ cm thick (the plates themselves are also about $1\frac{1}{4}$ cm thick). This ensures that the plate temperature is locally uniform. Large-scale uniformity is obtained by having six separate elements, each heating about 30 cm of the plate, on which the voltages can be varied independently. Because of the dependence of the local heat transfer on the inclination, adjustments have to be carried out until a surface temperature survey indicates uniformity each time the angle is changed. For the same reason uniformity cannot be obtained simultaneously above and below and changes must be made to the voltages on some elements when attention is transferred from one to the other. All the supplies derive from a constant voltage transformer. The maximum safe current in each element gives surface temperatures about 90 °C above room temperature.

The whole arrangement is mounted on a hinged frame and counter-balanced so that it can be set to any angle between vertical and horizontal. The traversing gear is attached to the plate so that it does not have to be readjusted when the angle is changed. On either side the traversing in the x-direction (which can be crude since the distances are large) is done by sliding along a rod the gear for the y-traversing. This consists of a screw traverse with attached scale enabling changes in the distance from the plate to be made and measured to 0·01 cm.

The angle of the plate is measured with a plumb-line attached to a protractor; this is nailed on to a board with one edge accurately parallel to the protractor edge so that both surfaces of the plate can be surveyed. Variations of the angle along the length of the plate are at their worst 15′.

4. Methods of measurement and observation

(*a*) *Surface temperature*

The temperature of the surface of the plate is measured with a Type M Stantel thermistor. This is supplied mounted on a small copper disk so that it can be put into intimate thermal contact with the surface. This has the advantage over any built-in thermometer that it can be traversed to any point on the plate, but the disadvantage that it can be calibrated only on a plate, and so only rather indirectly. The calibration has been made against a copper-constantan thermocouple on a heated copper plate (actually, a preliminary inclined plate apparatus); a junction very certainly at plate temperature could be produced by tapping down a constantan wire. The thermocouple was in turn calibrated against a mercuryin-glass thermometer in a water bath since the constitution of constantan varies too much between different manufacturers for tables to be used. A check that the thermistor calibration is satisfactory is possible using the manufacturer's formula $R = A\,e^{B/T}$, R being the thermistor resistance, T the absolute temperature, and A and B constants. It is a helpful coincidence that the variation of the unbalance current of a Wheatstone bridge containing the thermistor is almost linear with the temperature.

The surface temperature given by this method differs slightly, but systematically, from that indicated by extrapolation of the air temperature profile (measurement of which will be described shortly). The former has usually been 3–5 % higher in $T_1 - T_0$. The reason is unclear but the error is more likely to have been in the thermistor measurements than in the extrapolation; perhaps the thermistor reading is influenced a little by the air temperature as well as by the surface temperature. The disagreement is serious only where the temperature gradient at the surface is concerned. The thermistor method is satisfactory for indicating the temperature loading producing an observed velocity field.

(*b*) *Air temperature*

Platinum resistance thermometers have been used for all air temperature measurements. These are identical in design with the hot-wire anemometers standard in the Cavendish Laboratory, the current through them being low so that they are sensitive to temperature and not velocity. The sensitive portion consists of etched Wollaston wire $2{\cdot}5\mu$ in diameter and typically $0{\cdot}2$ cm long.

We are concerned, of course, with turbulent motion so the signal is analysed electronically. The averaging time usually needs to be as much as 10 min, indicating a pulse-counting method. Dr Townsend kindly lent me the apparatus he built for surveying the temperature field above a horizontal heated surface (Townsend 1959*a*). He has given a full description of this (Townsend 1959*b*) and I have used it without modification.

The mean temperature thus measured at various distances from the plate has been used to evaluate the gradient at the wall and so the convective heat transfer. (No internal device for measuring the total transfer is incorporated in the plate, as my impression from previous work is that it is not possible to correct satisfactorily for the radiative loss.) This has been done with a finite differences plot of the temperatures; $\Delta\overline{T}/\Delta y$ was plotted against y (taken midway between the two positions giving the finite differences) and extrapolated to $y = 0$. The plots naturally show considerable scatter, but this procedure keeps the general inaccuracy of gradient measurements down sufficiently to give $(\partial\overline{T}/\partial y)_{y=0}$ to about 10 %. Because of the difference, mentioned above, between the wall and air temperature measurements, only the latter have been used for this purpose.

One further complication should be mentioned. Δy between the wall and the first field point is known less accurately than between two field points. (The y-zeroing of a thermometer wire—and, incidentally, of a quartz fibre—is carried out by observing it and its optical image in the plate.). This may produce a small extra error in the extrapolations, but certainly not sufficient to explain the disagreement with the thermistor results. A check that $y = 0$ has been taken in about the right place can be obtained from the $\overline{T'^2}$ curve, since the fluctuations go to zero at the wall.

(c) *Temperature fluctuations*

Simultaneously with the mean temperature measurements, the mean-square fluctuation of the temperature $(\overline{T'^2})$ can be measured with Dr Townsend's pulse-counting apparatus. The arrangement is again described in Townsend (1959*b*). The apparatus is also equipped to measure other quantities, but with an inclined plate the introduction of two new variables—plate angle and distance from lower edge—clearly reduces the number of quantities that can be studied.

Since information about the velocity is not given in the form of an electrical signal, it is only the temperature fluctuations that can be analysed in this way. They thus provide the main source of information about the turbulence.

(d) *Velocity*

The velocity field is studied with quartz-fibre anemometers. An account of the reasons for using them and their design has been given in Tritton (1963*a*). My procedure for velocity surveys in the present project was to take spot readings of the position of the fibre end at 10 sec intervals, indicated by an auditory signal. A fibre would occasionally be in too rapid motion at the instant of the signal for a distinct position to be assigned to it; some averaging was thus involved in making the spot readings. The number of readings taken at each position was 20; this is certainly fewer than really desirable, but this method was used at a time when a large amount of rough information was of more value than a small amount of accurate data. The average of 20 readings, when plotted against position, shows sufficiently small scatter for a useful mean curve to be drawn.

For reasons given in Tritton (1963*a*) (fundamentally the non-linearity of the fibre response), a correction is needed when the mean velocity is determined from the mean deflexion. But the information necessary for the correction has not been obtained in any detail. The results will, therefore, be presented without correction; i.e. the velocities given will not be the true means, but values of $\tilde{u}$, obtained from $\bar{h}$ by reading from the (U, h)-curve (notation of Tritton 1963*a*) and related to the true mean by

$$f(\tilde{u}) = f(\overline{u}) + B\overline{u'^2},$$
$$f(u) = 8EIh/l^4 \simeq Au + Bu^2.$$

Whenever interpretation is made of the results, discussion will be given about the extent to which the correction might alter the conclusions. An appendix to this paper examines the size of the correction (which is small enough to allow this sort of treatment) and illustrates the procedure.

(e) *Simultaneous velocity and temperature studies*

In attempts to understand the physical mechanism of the flow, it is useful to have an indication of how the velocity and temperature fluctuations are correlated. With a quartz-fibre anemometer, there is no hope of measuring a rigorously defined correlation coefficient. However, it is very useful to have a device that gives simultaneously qualitative information about the velocity and the temperature. The information so obtained can include points that are in fact not contained in the formal correlation coefficient (e.g. the simultaneous occurrence of large velocity fluctuations and large temperature ones does not contribute to the coefficient if both remain independently distributed about the mean). Experimentally, this has been achieved by attaching a loudspeaker to Dr Townsend's resistance thermometer apparatus. This gives auditory information about the temperature fluctuations whilst a quartz-fibre anemometer is being observed. For example, the loudspeaker can be connected to receive the pulses from one of the trigger circuits so that it sounds whenever the temperature is greater than a fixed level. The pitch of the note gives some indication of the amount by which it is greater, but the more definite information about *when* it is greater is often sufficient.

(f) Smoke

Smoke visualization has sometimes been used as a supplementary method of elucidating the flow. It is not easy to introduce the smoke without serious disturbance, either in the form of buoyancy or of momentum, of the flow, but the use of ammonium chloride has largely overcome this difficulty. It is produced by bringing together the vapours of hydrochloric acid and ammonia. It is often satisfactory simply to hold the stoppers of the two bottles close to one another. For situations in which this would have had disadvantages, a device was set up to make ammonium chloride fumes drift in the required direction, with a speed too slow to disturb the flow. This consisted of a chamber with two small basins, one containing the acid and the other ammonia, set in its floor (a door separated these two when the device was not in use); a blower produced an elevated pressure in the chamber and the fumes were driven slowly out through very narrow slits.

5. Convection above an exactly horizontal surface

The motion above an exactly horizontal heated surface has been studied in some detail by other workers and is a useful starting point for considering the more complex situation when there is a small inclination; some of the processes controlling the convection are the same. The experiments that best reveal these processes are those in which the situation is designed to approximate to an infinite horizontal surface; the complication of a mean flow is then absent, and the mean heat transport is in the vertical direction. Except close to the plate, this transport is occasioned by turbulent fluctuations; these derive their kinetic energy entirely from the high potential energy of the unstable stratification. This situation does not involve any independent length-scale and, consequently the form of the dependence of the heat transfer on the temperature loading is predictable from dimensional considerations and is found to be

$$Q = \frac{H}{\rho C_p T} = \left\{\kappa g \left(\frac{T_1 - T_0}{CT_0}\right)^4\right\}^{\frac{1}{3}}.$$

C is dimensionless and taken to be constant (for constant Prandtl number), although in practice it has some dependence on T_1/T_0, because of variation of fluid properties with temperature. It was to minimize this that Thomas & Townsend (1957) worked in terms of $\log \overline{T}/T_0$ instead of $(\overline{T}-T_0)/T_0$. I am adopting this practice only when immediate comparisons with horizontal plate data are being made; elsewhere, it would make for complications (as when we consider the buoyancy force parallel to the wall, $g \cos\alpha(\overline{T}-T_0)/T_0$) and it should be noted that some of the results mentioned here were originally given in the logarithmic form.

The fact that Q is transferred across every plane parallel to the plate enables it to be used as a defining parameter of the problem. Following Thomas & Townsend (1957), scales of length, velocity and $(T-T_0)/T_0$ are then respectively:

$$y_0 = \left(\frac{\kappa^3}{Qg}\right)^{\frac{1}{4}}, \quad u_0 = (Qg\kappa)^{\frac{1}{4}}, \quad \theta_0 = \left(\frac{Q^3}{\kappa g}\right)^{\frac{1}{4}}.$$

In experimental approaches to this problem there has been, as might be expected, some controversy about what arrangement does produce a convection pattern most closely approximating to the infinite plate case. The prevention of inflow from the sides by having walls seems highly desirable, but it has been objected (Croft 1958) that this results in a strong circulatory motion. Thomas & Townsend (1957) and Townsend (1959*a*) thought this insufficient to alter the results and they give much detailed information about the turbulence in a box with a heated bottom and open top.

Townsend concludes that there are two quite sharply distinguished modes of temperature fluctuation, 'active' and 'quiescent'. The name of the latter is appropriate only so far as temperature fluctuations are concerned, since, during such a period, there are velocity fluctuations at least as intense as during an active one, though of smaller scale. The active periods, which are thought to control many of the main features of the convection (e.g. the mean temperature profile), result from localized up-draughts of hot air originating close to the surface. Such 'plumes' were shown clearly by Ramdas & Malurkar (1932) who photographed the 'steam' produced by spreading a thin water film on a heated plate; there is also evidence for them in the Schlieren photographs taken by Schmidt (1932) and Weise (1935).

The mean temperature profile is of particular interest, because it indicates the longitudinal buoyancy force when the plate is inclined. Sufficiently far from the surface ($y > 8y_0$ perhaps), it may be taken in the form

$$(\overline{T}-T_0)/T_0 \propto y^{-n}, \tag{1}$$

but there is some dispute as to the value of n. The uncertainty arises partly from the fact that the same data can be fitted to different values of n by different choice of T_0. And it is not always clear what is the correct value—complicated behaviour at the top of a laboratory apparatus may cause the profile to asymptote to a temperature higher than room temperature.

Taking T_0 close to the temperature at the top of their box, Thomas & Townsend conclude that $n = 1$. This is the same as the value given by Malkus's (1954) theory (though there is not complete agreement in the constant proportionality). On the other hand, it is in conflict with the prediction, $n = \frac{1}{3}$, of similarity theory

(Priestley 1954); this is simply dimensional analysis based on the supposition that the profile does not depend on κ, in other words that, as in shear flow, molecular effects do not influence the large-scale properties of the turbulence. Priestley claims that meterological data support the $n = \frac{1}{3}$ profile. It is unfortunately seldom that such data are free from the effect of wind shear, and Townsend (1959*a*, 1962) claims that more recent meteorological experiments (Webb 1958) show n increasing above $\frac{1}{3}$ when this is very weak. $n = \frac{1}{3}$ has not been obtained in any laboratory set-up, but Croft (1958) suggests $n = \frac{1}{2}$. He used a small plate (10×8 cm) with no side-walls, and it is hard to believe, in view of the observations of Weise (1935) and Kraus (1940) with very similar arrangements, that there was not a considerable variation of the profile with position on the plate, produced by inflow at the sides to a permanent plume rising off the plate.

6. General features of inclined plate flow

When the plate is inclined, it is convenient to think in terms of directions relative to the plate rather than relative to the horizontal and vertical and, correspondingly, to use Cartesian co-ordinates as defined in §2. The buoyancy force then has components both parallel and normal to the plate, but these affect the flow in very different ways. The wall blocks any general motion in the net direction of the buoyancy force (vertically upwards) and so only the longitudinal component produces a mean motion.

The normal buoyancy force produces an overturning motion similar to that above a horizontal surface. It is helfpul to think of this as maintaining itself more-or-less independently of the mean motion and governing the mean temperature distribution and the structure of the turbulence. The extent to which this is a valid assumption will be seen in subsequent sections, but it is immediately plausible when, as throughout this paper, the angle of the plate with the horizontal is small. Then y_0, u_0 and θ_0 may be kept as scales of length, velocity and $(T-T_0)/T_0$; but they now have to be written

$$y_0 = \left(\frac{\kappa^3}{Qg\sin(-\alpha)}\right)^{\frac{1}{4}}, \quad u_0 = \{Qg\kappa\sin(-\alpha)\}^{\frac{1}{4}}, \quad \theta_0 = \left(\frac{Q^3}{\kappa g\sin(-\alpha)}\right)^{\frac{1}{4}},$$

so that they are determined by the normal buoyancy force.

This way of scaling will be extended to include the use of x/y_0 as the non-dimensional distance from the lower edge. This is appropriate when the development of the mean flow is governed by the thermal turbulence. Its relationship to the Grashof number, defined as

$$\mathrm{Gr} = \frac{g\cos\alpha}{\nu^2}\frac{T_1-T_0}{T_0}x^3$$

should be recorded, to relate this approach to the more general one applicable for all α. It is

$$(\mathrm{Gr})^{\frac{1}{3}} = \frac{x}{y_0}\left[\frac{C\cot(-\alpha)}{(\mathrm{Pr})^2}\right]^{\frac{1}{3}},$$

where C is the heat-transfer coefficient already defined for the horizontal plate and to be introduced shortly for the inclined. For the arrangement described in §§7–12,

$$(\mathrm{Gr})^{\frac{1}{3}} = 1{\cdot}1x/y_0.$$

The flow up the plate produced by the x-component of the buoyancy force has a boundary-layer character. This statement will be qualified in §11, but in general the experiments confirm it. It is to be expected from the fact the buoyancy force, as produced by a temperature profile resembling that above a horizontal surface, is large only very close to the plate; also from our knowledge that in related situations free convection produces boundary-layer flows. The governing equations may thus usefully be formulated with the boundary-layer approximation. No mathematical theory of this flow is possible, but the mean velocity equation is still of particular interest for the information provided by a comparison of the sizes of the various terms (see §11). This equation is

$$\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y} = g\cos\alpha\left(\frac{\overline{T}-T_0}{T_0}\right)-\frac{\partial}{\partial y}\left(\overline{u'v'}-\nu\frac{\partial \bar{u}}{\partial y}\right). \tag{2}$$

It suffices here to say that this is subject to the usual approximations; full discussions of the equations of convective flows are given by, amongst others, Squire (1953), Calder (1949) and Tritton (1960).

It is worth mentioning here an unsuccessful attempt to treat this equation theoretically. The reader may then appreciate better why I undertook this investigation—the interesting results of which were not ones that could be anticipated. Also some of the interesting points are precisely the reasons why the assumptions of the theory fail.

The idea was that the mean temperature profile and the structure of the turbulence could be used to determine the otherwise unknown terms in (2), on the supposition that they are unaltered from the horizontal plate convection. The buoyancy-force terms could be inserted straightaway. For the Reynolds stress, the assumption was made that the eddy viscosity was equal to the eddy conductivity that can be defined from the heat transfer and the temperature distribution. Then, a prediction of the development of the boundary layer up the plate was possible. The interest was that the three values of n (the exponent in (1)) gave very different predictions; with $n = 1$, the boundary-layer thickness decreased with increasing x, but with $n = \frac{1}{3}$ or $\frac{1}{2}$, it increased. An inclined-plate experiment thus seemed to offer a possible way of settling the controversy about the value of n—a point of some importance in view of the different predictions of Malkus's theory and similarity.

A preliminary smoke experiment gave a strong indication that the boundary layer did have the unusual (but, in free convection, not impossible) feature, predicted by considering the Thomas & Townsend profile, of decreasing in thickness. It will be seen in §11 that this observation was in sense correct, but that the decrease was part of a much more complex behaviour.

The attempted theory was carried further and, of the various results, there is some interest in the prediction that the velocity maximum was close to the wall, little further out than the edge of the viscous sublayer. The experiments (see §11) show it to be much further out than that. This is one of the ways in which the difference between actual behaviour and the assumptions of the theory is demonstrated.

7. Experimental arrangement; further general features of the flow

I have made a detailed study of the temperature and velocity fields with the small inclination of $9^\circ\,47'$ ($\alpha = -80^\circ\,13'$) and a plate temperature of about 57 °C above room temperature.

The flow depends appreciably on the upper end conditions. This complication arises through a tendency for the permanent plume that rises off the centre of

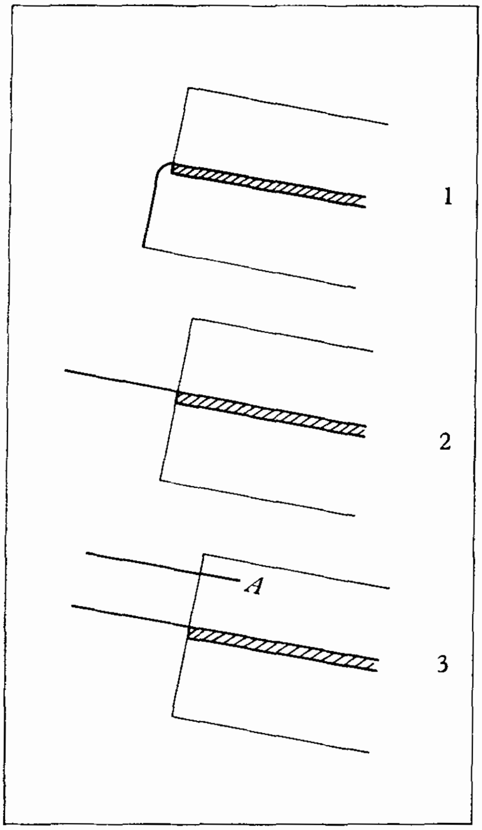

FIGURE 1a

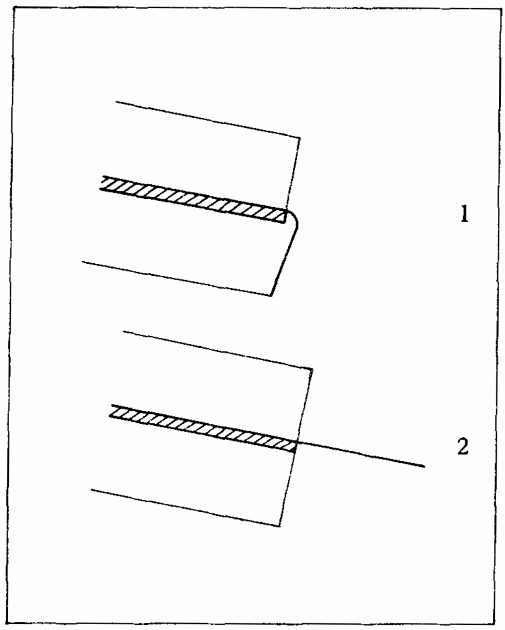

FIGURE 1b

FIGURE 1a. Upper end arrangements. Thick lines show cross-section at any z; thin lines the edges of the side-walls, into which the screens fitted tightly. Diagrams are approximately to scale, being about $\frac{1}{30}$ actual size.

FIGURE 1b. Lower end arrangements shown in the same style as figure 1a.

the plate when it is horizontal to retain its influence when there is a small inclination. For an inclination of about 4°, for instance, there is an inflow at the upper end not very different from that above the horizontal plate. As the angle is increased the plume is centred on an increasingly higher x, and at the angle chosen for the detailed investigation it rises sufficiently near the top of the plate for most of the flow to be regarded as a boundary-layer flow up the plate. These comments derive both from smoke experiments and from brief surveys of the velocity field.

However, the exact position of the plume and consequently the details of the velocity profile some distance upstream of it (to about the middle of the plate) depend strongly on the conditions at the upper end. Figure 1*a* shows various screening arrangements for which the flow was examined. Each, of course, had as one of its purposes the prevention of hot air from the underside of the plate coming into the plume. In figure 2 are plotted a few velocity measurements at $x = 114{\cdot}4$ cm with each arrangement; measurements at $81{\cdot}0$ cm are also given to show that there is negligible effect there. It is clear that the ability of air to enter from below in case 1 allows the plume to come down to a lower x.

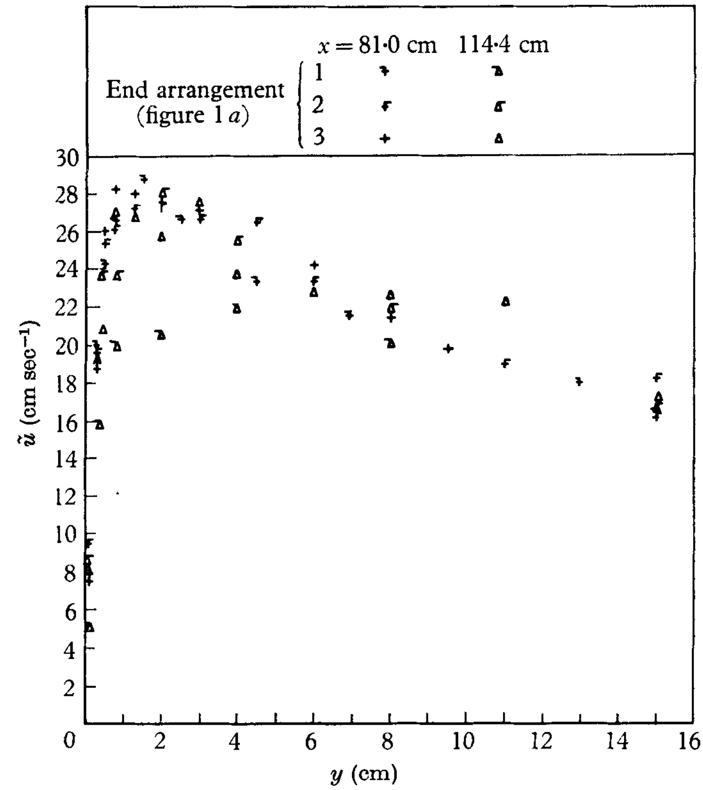

FIGURE 2. $\tilde{u}$-profiles for the different upper end conditions.

The arrangement used during the detailed survey was number 3. The upper screen served to inhibit the plume and smoke experiments indicated that most of the air in the boundary layer continued to move in about the same direction after getting beyond the end of the heated plate. Smoke introduced at A (figure 1*a*–3) sometimes moved into the space between the screens but more often rose above the upper screen, the whole behaviour being highly intermittent. This shows that the channel between the screens did not act as a chimney and draw air into it; there was still a weak plume rising with its centre roughly above the end of the heated plate.

In view of these observations, I felt that this arrangement gave a better approximation than any other to a semi-infinite plate. Surveying of the velocity and temperature fields was largely restricted to the region in which velocity profiles given by arrangements 2 and 3 (figure 1a) were little different.

Lower edge conditions are much less critical. No effect could be observed on changing between the two arrangements shown in figure 1b. It is necessary only to prevent hot air from below the plate entering the flow above. The arrangement actually used during the detailed surveying was no. 1.

All the surveying of the temperature and velocity fields was carried out approximately on the centre plane in the z-direction, in the hope that symmetry would make the assumption of two-dimensionality involved in equation (2) not too far from the truth.

8. The temperature field

A full survey of the temperature field above the plate was made, with the principal purpose of discovering how closely the supposition that it was little affected by the inclination was justified. Accurate averages were obtained with Dr Townsend's pulse-counting apparatus (§ 4 (b)).

x (cm)	C
19·2	3·45, 3·35
39·8	3·5, 3·3
60·5	4·1, 3·9
82·2	3·35, 3·2
103·6	3·35, 3·05

TABLE 1

Figures 3a and b show the resulting mean-temperature profiles for respectively low and high y. Marked on the latter are readings taken at a height roughly equal to that of the side-walls. The general trend is for the temperature here to increase with x, and it is clear from figure 3 that each profile asymptotes more nearly to this than to the room temperature (or any other independent of x).

When this has been allowed for, there is a close resemblance between these profiles and those above a horizontal plate given by Townsend (1959a). A curve representing his results is included in figure 3a (at low y the influence of the differing effective zeros is small and all the profiles may be compared with the one line). There is no systematic variation of the heat transfer with x; values of C, defined as

$$C = \theta_0^{-1} \log \frac{T_1}{T_0} = \left(\frac{\kappa g \sin(-\alpha)}{Q^3} \right)^{\frac{1}{4}} \log \frac{T_1}{T_0},$$

are given in table 1. The first of each pair of values is calculated with T_0 taken as room temperature; the second with T_0 as the asymptotic temperature at that x. Thus constancy of the former indicates constancy of the heat transfer (apart from corrections for changes in room temperature and pressure), while the latter probably provides the better comparison with the horizontal plate. Values of

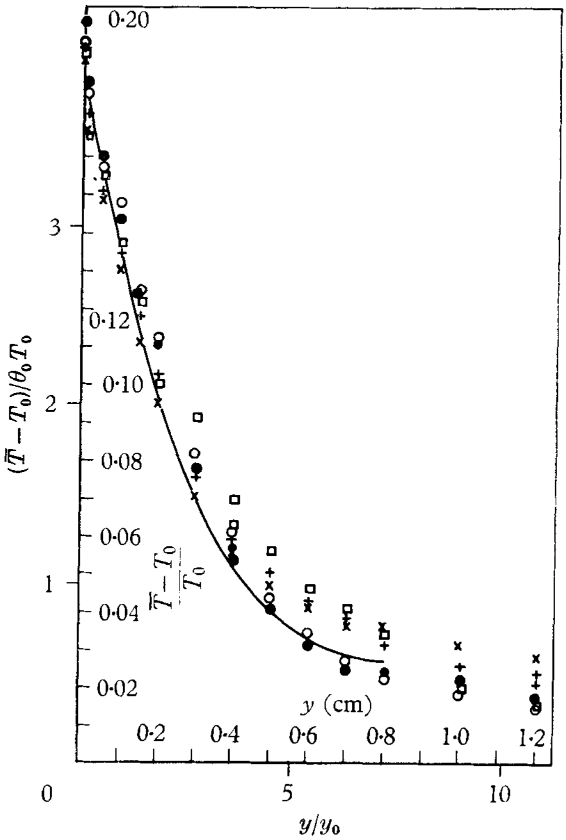

FIGURE 3*a*. Mean temperature profiles for lower range of y. Values of y_0 and θ_0 are defined in the text. The line is derived from the horizontal-plate data of Townsend (1959*a*). ●, x = 19·2 cm; ○, x = 39·8 cm; □, x = 60·5 cm; +, x = 82·2 cm; ×, x = 103·6 cm.

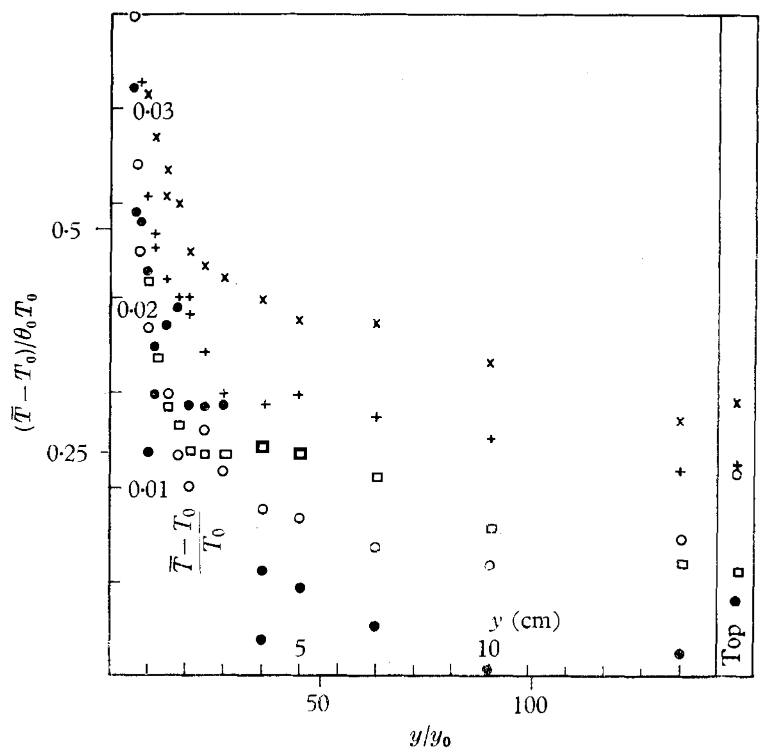

FIGURE 3*b*. Mean temperature profiles for higher range of y. The points in the column marked Top indicate mean temperature measurements level with the top of the side-walls ($y \sim$ 30 cm).

C given for the horizontal plate are 3.55† (Thomas & Townsend 1957), 3·4 (Croft 1958) and 3·28† (Townsend 1959*a*).

The scales y_0, u_0 and θ_0 may now be evaluated for the inclined-plate flow and used in the presentation of further results. Their values have been taken as 0·111 cm, 2·26 cm sec^{-1}, and 0·0474.

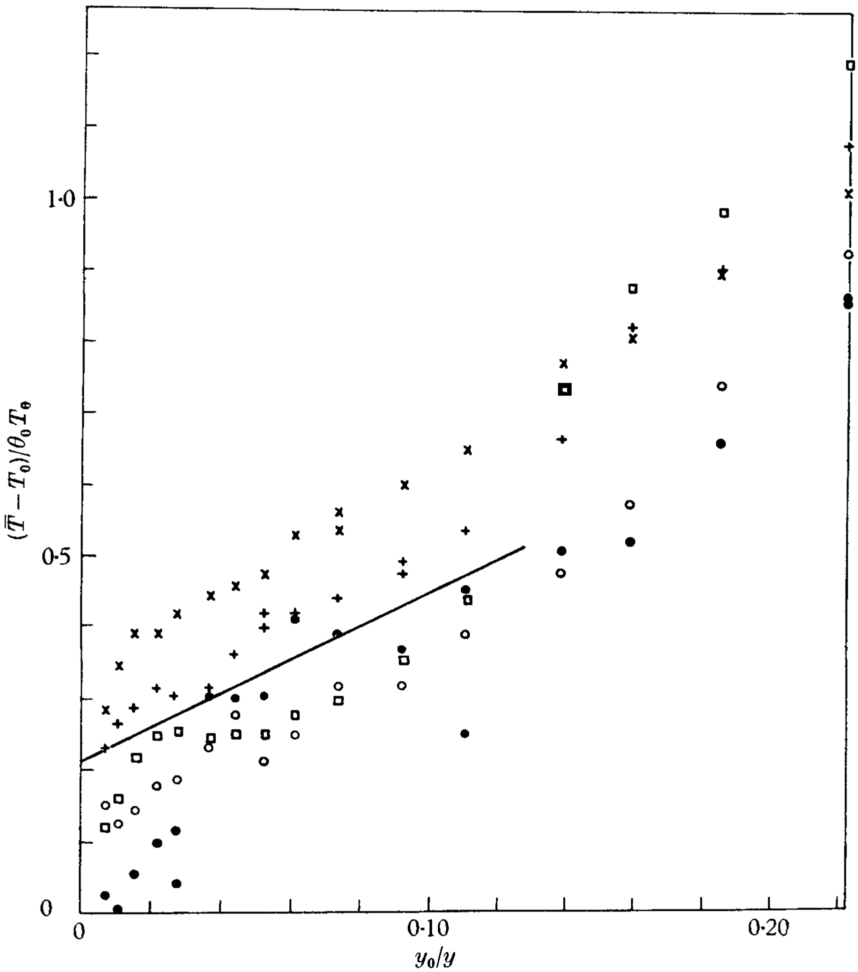

FIGURE 4. Mean temperatures in the outer region plotted against $1/y$. The line indicates the gradient of the horizontal-plate data of Townsend (1959*a*). ●, $x/y_0 = 173$; ○ = 358; □ = 545; + = 741; × = 934.

The similarity at large y to Townsend's results for a horizontal plate is best shown by plotting $(\bar{T}-T_0)/\theta_0 T_0$ against y_0/y as in figure 4; this eliminates the difficulty about the choice of a zero. The line in figure 4 has a slope of 2·37, the value for the horizontal plate‡ (for $y/y_0 > 8$). Figures 5 and 6 show $(\bar{T}-T_0)/\theta_0 T_0$

† The figures actually quoted by Thomas & Townsend and Townsend are respectively 3·4 and 3·14, but they appear to have been calculated using the conductivity at ambient temperature, whereas here we use the value at $(T_1 T_0)^{\frac{1}{2}}$. Croft appears to have followed a similar practice.

‡ From Townsend's 2·6 with different choice of κ.

plotted against $(y_0/y)^{\frac{1}{3}}$ and $(y_0/y)^{\frac{1}{2}}$. The problem of selecting the best exponent is well illustrated. It may be even more difficult than in the horizontal-plate case; it will be seen in § 11 that the mean flow field is sufficiently complex for some of the apparently unsystematic deviations in the mean profiles to be due to convection effects associated with the term $(\overline{\mathbf{u}}.\mathrm{grad}\,\overline{T})$ and not just to experimental error. Except at low x, repeated readings suggest that the latter is too small to provide a complete explanation.

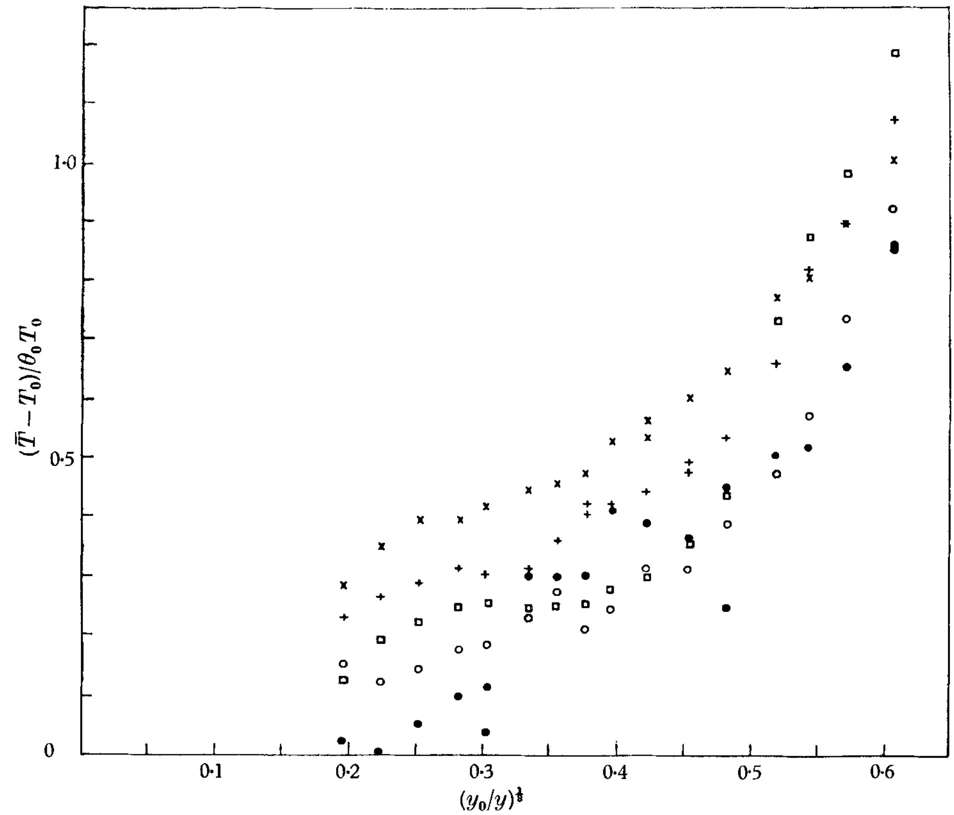

FIGURE 5. Mean temperatures in the outer region plotted against $y^{-\frac{1}{3}}$. Values of x/y_0 for the symbols are as in figure 4.

Comparison of figures 4 and 5 suggests that at the largest y, a $y^{-\frac{1}{3}}$ profile might represent the points better than a y^{-1} one, though only at the cost of taking T_0 *lower* than room temperature for some values of x. Departure from this representation would begin for $y/y_0 < 11$; the y^{-1} profile is a good approximation for all $y/y_0 > 8$. Clearly in these circumstances, a $y^{-\frac{1}{2}}$ representation is also possible; figure 6 demonstrates this but also shows that the constant of proportionality would have to be quite different from that given by Croft.

The general conclusions are thus as follows. There may be a greater tendency than in the horizontal plate case for 'similarity' (independence of κ) to apply at the largest y, but the profile is really very little altered from the one found by Townsend for that case. It should be noted that the experiment must *not* be interpreted as an independent check on Townsend's work; since the technique was identical with his, it could obviously not be such. Its purpose was otherwise—to discover how much the inclination changed the profile (and comparisons with suggestions other than Townsend's have been made to see whether any change was towards these). Whether or not the best value of n (the exponent in (1)) remains 1, it still appears that at every y/y_0, $(\overline{T}-T_0)/\theta_0 T_0$ is close to its value for a horizontal plate. Since it will next be seen that the convection *is* appreciably changed in other respects, this is a significant fact.

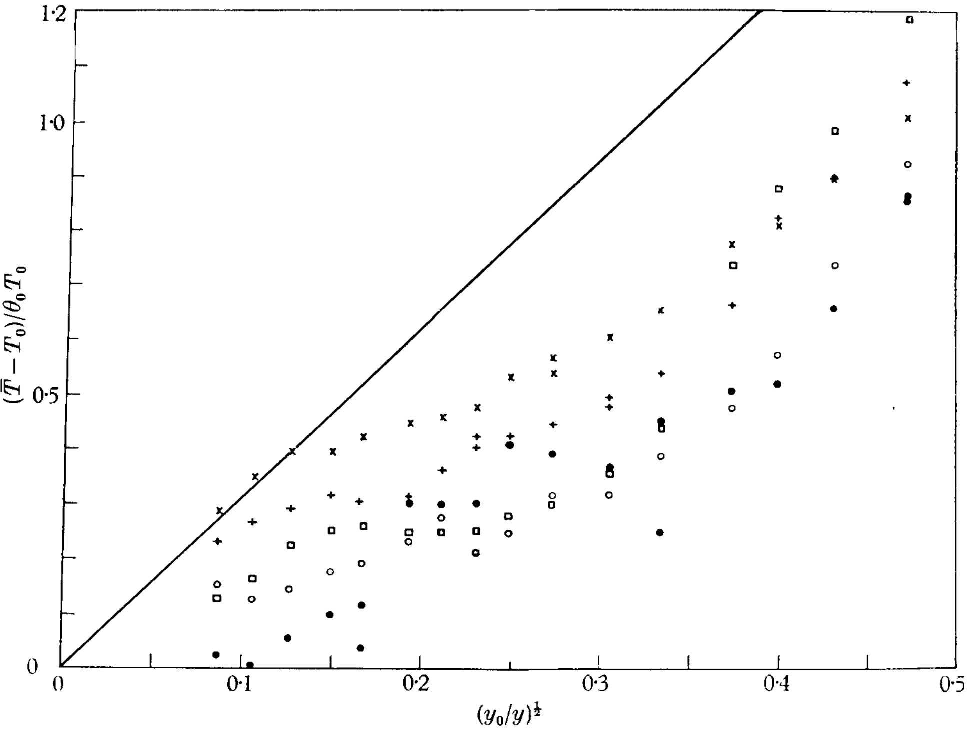

FIGURE 6. Mean temperatures in the outer region plotted against $y^{-\frac{1}{2}}$. The line corresponds to the horizontal-plate data of Croft (1958). Symbols as for figure 4.

9. Temperature fluctuations

Simultaneously with the mean temperature measurements, information was obtained about the temperature fluctuations. Qualitatively it was evident that at all x the 'active plumes' of the horizontal-plate convection (see §5) were occurring. Asymmetry of the temperature distribution was evident in the behaviour of a monitor galvanometer for $y/y_0 > 4$, and when $y/y_0 > 10$ the predominant features were that the temperature stayed for most of the time at an only weakly fluctuating low temperature and that there were sharp 'excursions' to much higher temperatures. For $y/y_0 < 1{\cdot}5$, there was evidence of the reverse behaviour—asymmetry being produced by excursions to low temperatures.

The quantitative study of the fluctuations consisted of measurements of $\overline{T'^2}$. The results are plotted in figures 7*a* and *b*; the line represents Townsend's (1959*a*) horizontal-plate data.

Systematic variations with x are to be detected from these plots. Considering any pair of them, the one at larger x rises more steeply from the origin to a maximum that tends to be higher and at a smaller y/y_0; though in the immediate neighbourhood of the maximum the trends are not so distinctive, probably because the increase in maximum $(\overline{T'^2})^{\frac{1}{2}}/\overline{T}\theta_0$ is more marked at lower x and the shift to lower y/y_0 more marked at higher x. Outside the maximum the curve for larger x, of any pair of curves, falls off more rapidly than the one at smaller x, so that there

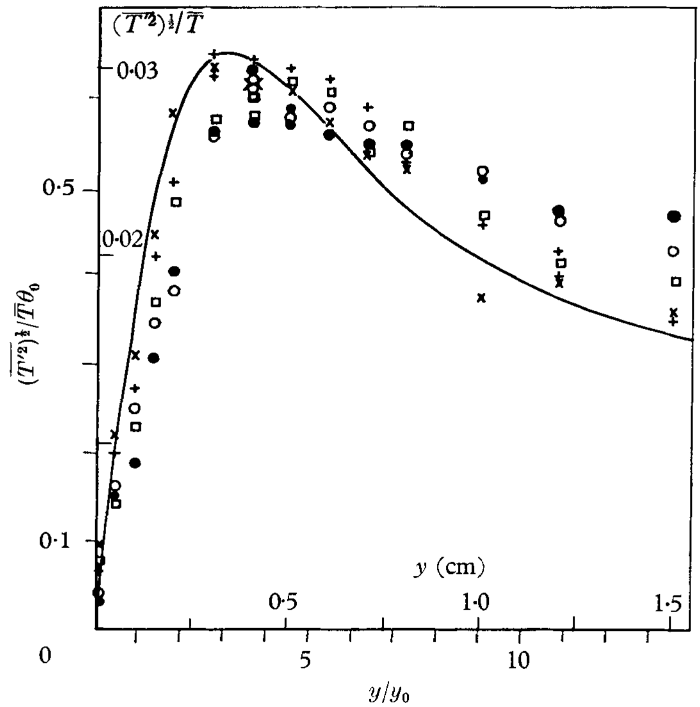

FIGURE 7*a*. Intensity of temperature fluctuations for lower range of y. Symbols are defined in figure 4. The line represents the data for a horizontal plate (Townsend 1959*a*).

is a region (roughly $9 < y/y_0 < 40$) in which $\overline{T'^2}$ decreases with increasing x at constant y. The final fall-off at large y is, on the other hand, more rapid at the smaller x, so that the trend with x is again reversed for roughly $y/y_0 > 70$. These features are illustrated by figure 8 which shows $(\overline{T'^2})^{\frac{1}{2}}/\overline{T}\theta_0$ plotted against x/y_0 for various constant values of y/y_0. These values have, of course, been chosen to show the systematic effects clearly; this does not mean that they are not representative of all values of y/y_0 in their vicinity, but the variations are naturally not well illustrated by such plots in regions where the trend is changing from $\overline{T'^2}$ increasing with increasing x to its decreasing.

Despite the complexity of these changes the trend throughout† is for the curve to become closer to that for the horizontal plate as x is increased. The distribution of $\overline{T'^2}$ is developing towards the horizontal plate distribution as the

† Except possibly very close to the wall ($y/y_0 < 1{\cdot}5$), where this is still the trend for the lower values of x but there seems to be some 'overshooting' at the higher.

flow proceeds up the plate, having commenced near the lower edge noticeably different. Observations to be described in the next section help with the interpretation of this result.

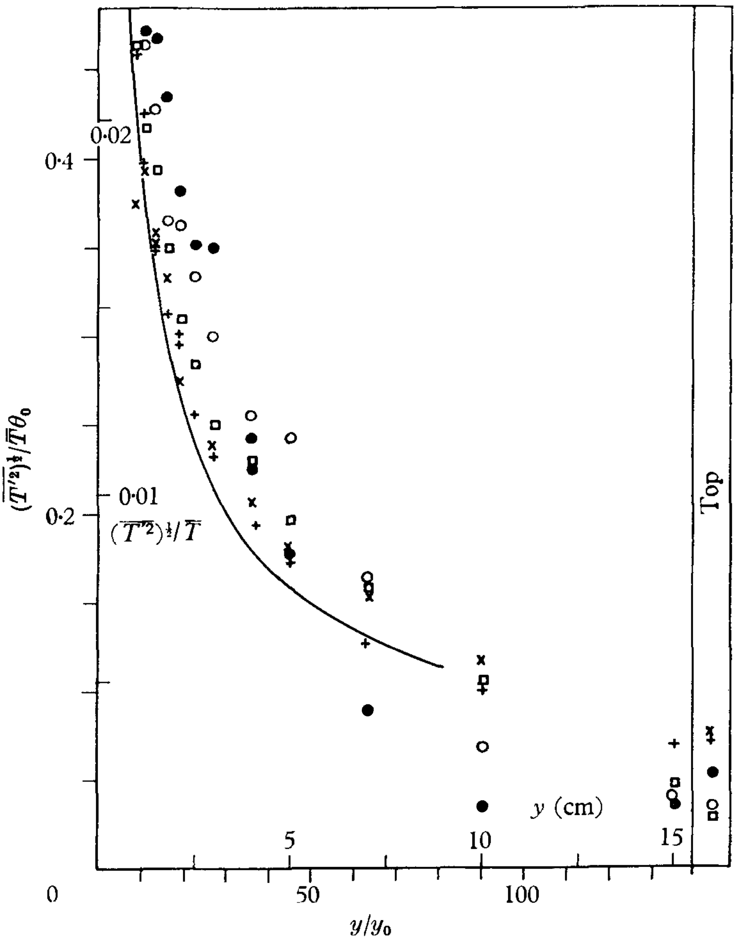

FIGURE 7*b*. Intensity of temperature fluctuations for higher range of y, plotted in the same style as figure 7*a*. The points in the column marked Top indicate measurements level with the top of the side-walls.

10. The development of the small-scale turbulence

Before coming to the details of the velocity field, it is of interest to consider one qualitative observation that became apparent when a fibre anemometer was introduced at various distances from the lower edge and that is surely connected with the phenomenon just described. The quantitative form of this observation is that the mean-square deflexion $\overline{h'^2}$ for a fibre (particularly a short one) is lower at small x than at large. Closer inspection of the motion of the end of a fibre reveals that this comes about in rather a special way. Over the lower part of the plate ($x/y_0 < 700$ perhaps, though obviously the division is not sharp) there are periods during which the flow is in effect laminar; the fibre remains still between sharply contrasted periods of activity. Moreover, the position in which it remains still is the same each time indicating that the velocity is the same during all the laminar periods, or more precisely, the variations are small compared with the 'active' fluctuations. This behaviour is not to be observed further up the plate.

Simultaneous velocity and temperature fluctuation studies (§4 (*e*)) showed that the laminar periods are concurrent with temperature quiescence (in the sense that Townsend 1959*a* used the word for the horizontal plate—see §5). The details of the observations (made at $x/y_0 \sim 190$) were as follows. With the fibre and wire close to one another at both $y/y_0 \sim 18$ and ~ 70, there was strong

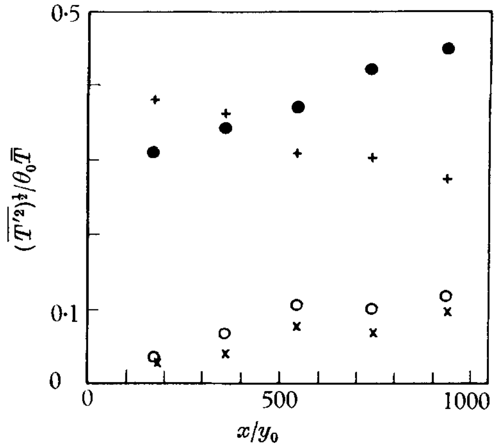

FIGURE 8. Variation with distance up the plate of the intensity of temperature fluctuations at selected values of y. ×, $y/y_0 = 0{\cdot}09$; ● = 1·35, + = 18·9, ○ = 90·2.

correlation between the temperature being high and the fibre being in vigorous motion. Very occasionally temperature activity occurred at $y/y_0 \sim 18$ without associated velocity activity. Rather more markedly, the reverse was possible at $y/y_0 \sim 70$; but when the fibre was left in this position and the wire traversed back to $y/y_0 \sim 30$ fibre activity was always associated with temperature activity, implying that the fluctuations not having a high temperature at the same position were the residual motions of a plume not reaching that position. With the fibre at $y/y_0 \sim 1{\cdot}8$ and the wire at ~ 18 there was again clear association between temperature and velocity activity; when the former was occurring, the velocity tended to be particularly high or particularly low, indicating that a large part of the fluctuations at low y resulted from inflow to the plumes.

It may be concluded that the large-scale plumes are occurring throughout, but over the lower part of the plate, the surrounding fluid is not in the state of small-scale turbulence that is characteristic of the horizontal-plate convection. Contrastingly, at higher x (full observations were made at $x/y_0 \sim 700$), there is little or no difference in the fibre motion during temperature activity and quiescence, which, in view of the results in Tritton (1963*a*), suggests that the small-scale motion is more intense.

These observations may be interpreted without reference to the shear of the mean flow, and the development of the turbulence as the air flows up the plate may be likened to its development over a horizontal plate suddenly brought to a high temperature. The large-scale plumes would start occurring immediately, being affected only in their detailed properties by the stillness of the surrounding fluid. But the small-scale turbulence, deriving its energy from the dissipation of the plumes, would take some while to attain its steady-state intensity. The variation in the $(\overline{T'^2})^{\frac{1}{2}}/\overline{T}\theta_0$-curve with x described in § 9 can now be seen to result from this development. The differences between the low x and horizontal plate curves show the effect of the small eddies.

In view of these observations the similarity of the mean-temperature profiles at different distances up the plate is a clear demonstration that the profile is mainly a consequence of the large-scale motions.

11. The velocity field

The information about the velocity field is unfortunately not capable of clear-cut presentation. Certain general, and interesting, conclusions are plainly inducible, but the detailed material is somewhat amorphous. There are several reasons for this. (i) The measurements are of $\tilde{u}$ and not $\overline{u}$, and, since the conversion from one to the other is uncertain, it seems preferable to keep them in this form. (ii) The behaviour of the flow has proved to be more complex than expected and the time available for the work was limited. The experiments were thus planned to give as much information as possible at the cost of this information being rather rough. (iii) The details of the flow varied between different runs, probably because of changes in the general circulation in the room.

A full survey of the mean velocity field has been carried out by the method of taking 20 spot readings with a fibre anemometer as described in § 4 (d). A briefer survey with a second anemometer (of different length and diameter) was made as a check on the main results.

In figures 9a–c, the development of the mean velocity field up the plate is shown by a series of plots of $\tilde{u}$ against y/y_0. There is one profile common to figures 9a and b and similarly with 9b and c, so the full variation with x can be traced in these diagrams. Two complications, one merely tiresome, the other of physical interest, are evident from these results with the consequence that the theoretical treatment mentioned in § 6 does not describe the situation. First, there is a large inflow of air across the $x = 0$ plane and the velocity boundary layer develops on top of this, and, secondly, it is not a good approximation to equate the eddy conductivity and viscosity. The former does, in a sense, produce a boundary layer of decreasing thickness, since the range of y over which the velocity is close to its maximum value diminishes.

The inflow at the bottom of the plate extends the full height of the side-walls and there is a velocity of the same order, though with some variation with x, at this height all the way up the plate. Figure 9 includes measurements of this, and it is clear that the velocity layer has this, rather than zero, as its outer boundary condition. This 'through-flow' probably results from the elevated temperatures between the side-walls at large y having a chimney action. A rough calculation of the pressure difference between the ends of the plate produced by the observed excess of asymptotic temperatures over room temperature suggests that this could produce without friction a velocity of 30 cm sec^{-1}, to be compared with observed through-flows of around 20 cm sec^{-1}.

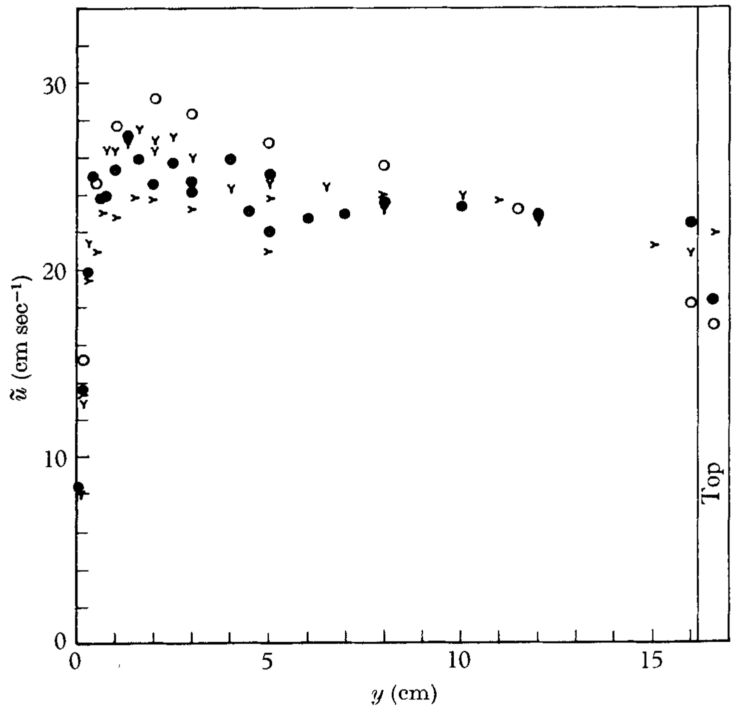

FIGURE 9*a*. $\tilde{u}$-profiles for lowest range of x. The column marked Top indicates measurements level with the top of the side-walls. ≻, x = 7·0 cm; ●, x = 19·0 cm; Y, x = 27·6 cm; ○, x = 43·2 cm.

A possible explanation of the occurrence of the elevated temperature and so of the chimney effect may be arrived at by considering what happens at the top of the side-walls. In Townsend's horizontal plate apparatus the final mechanism of heat transfer was a permanent plume above the side-walls. When the plate is inclined, the development of the velocity boundary layer could produce an opposing influence on the air level with the top of the side-walls—and, indeed, a smoke experiment indicated that this air does move downwards more frequently than upwards, in a highly intermittent overall behaviour. Hence, the permanent plume mechanism is probably no longer possible. Instead, the through-flow combined with the increase in asymptotic temperature with x, can result in the final direction of heat transfer being parallel to the plate. Rough calculations suggest that most of the heat lost by the plate is indeed carried off by the through-flow; that is to say

$$\int \frac{U}{T_0}\frac{\Delta T}{\Delta x}\,dy$$

(integrated over the height of the side-walls) equals at least 0·8Q.†

† The extent to which this necessitates reinterpretation of the temperature field will be considered in §14.

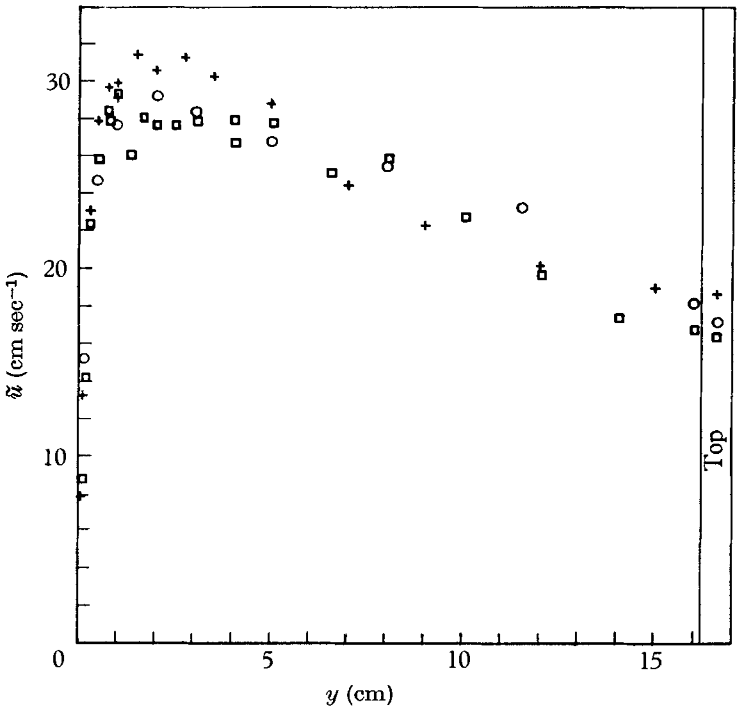

FIGURE 9b. $\tilde{u}$-profiles for middle range of x. One profile of figure 9a is repeated. The column marked Top indicates measurements level with the top of the side-walls. ○, x = 43·2 cm; □, x = 61·6 cm; +, x = 82·4 cm.

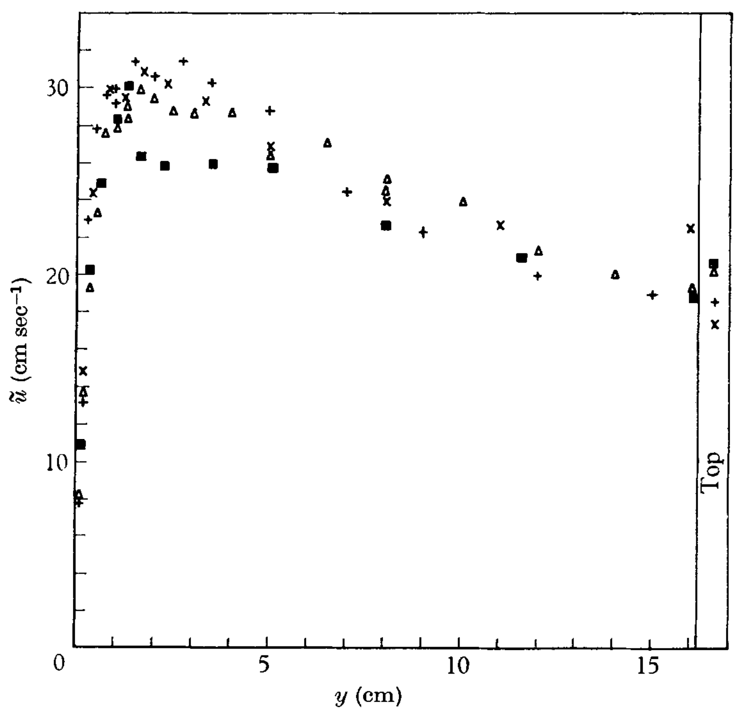

FIGURE 9c. $\tilde{u}$-profiles for highest range of x. One profile of figure 9b is repeated. The column marked Top indicates measurements level with the top of the side-walls. +, x = 82·4 cm; ×, x = 98·6 cm; △, x = 117·0 cm; ■, x = 141·1 cm.

The details of the through-flow are of little interest, as they undoubtedly depend on the length and width of the plate and the height of the side-walls. In fact, they even changed a little between the main survey and the subsidiary one with a different fibre (figure 10); the difference in the velocity of the through-flow is larger than can be explained either by errors in fibre calibration or by the different relationships between $\tilde{u}$ and $\bar{u}$. The details thus presumably also depend on the general circulation of air in the room, this being produced mainly but not entirely by the heated plate (it is probable that the radiators in the room were hot during the main survey but cold during the subsidiary one). On the other hand, the *occurrence* of a through-flow is, surely, an invariable feature.

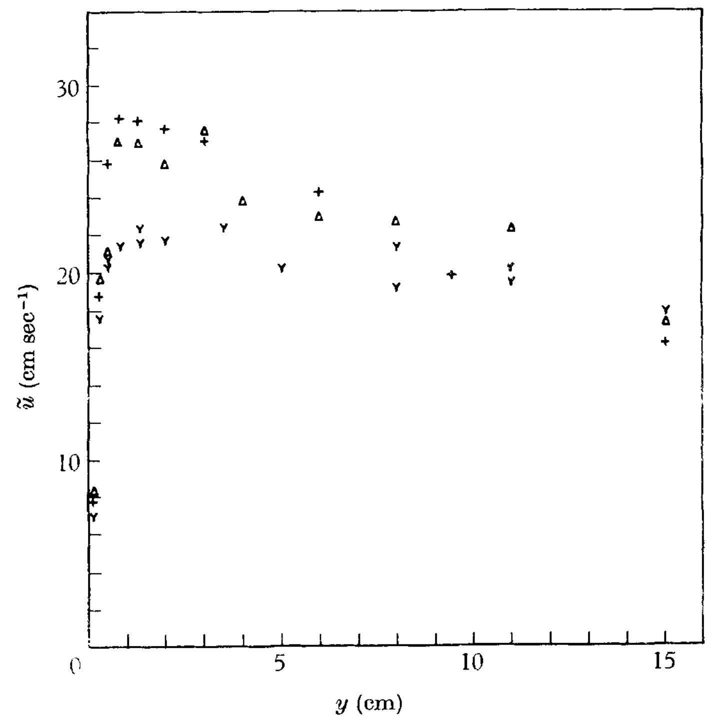

FIGURE 10. $\tilde{u}$-profiles; subsidiary survey. Y, x = 27·6 cm; +, x = 81·0 cm; △, x = 114·4 cm.

Despite the implication of these conclusions that the flow does not satisfy the usual suppositions (and, incidentally, that u_0 is unlikely to be the sole velocity scale), an examination of the development of the boundary layer by the buoyancy force leads to some interesting conclusions. The velocity maximum is surprisingly far from the wall, where the buoyancy force is small. There is evidence that the acceleration of the air in the region of the maximum is greater than could be produced by the local buoyancy force, and thus that turbulent transport of momentum to this value of y must occur, which means transport against the gradient. It is convenient to consider the acceleration right at the maximum, as the $\bar{v}\,\partial\bar{u}/\partial y$ part of the inertia can then be left out of consideration, and the fact that $y_{\max}$ changes little with x (see figure 9) allows finite differences in $u_{\max}$

to be used to determine $\bar{u}\,\partial\bar{u}/\partial x$. It then seems that this exceeds $g\cos\alpha(\bar{T}-T_0)/T_0$ in the region $0 < x < 80\,\text{cm}$ ($\sim 700\,y_0$).

Such a conclusion is sufficiently surprising that one would like the evidence for it to be strong. Because of the experimental difficulties, the possibility remains open that there is some other explanation of the observations, not involving this conclusion.

One of the problems is the difference between $\tilde{u}$ and $\bar{u}$, making it dangerous to draw conclusions involving velocity differences directly from the data of figure 9. Consideration is given to the possible size of this difference in the appendix, but if it is grossly under-estimated the conclusions would have to be modified.

Secondly, there is the problem of what value to use for T_0 in evaluating the buoyancy force. The excess of inertia over buoyancy is marked if it is taken as the value approached by $\bar{T}$ at large y. A case for taking it as this may be advanced on the grounds that buoyancy resulting from the elevated asymptotic

x (cm)	$\dfrac{g\cos\alpha(\bar{T}-T_a)}{T_a}$ (cm sec^{-2})	$\dfrac{g\cos\alpha(\bar{T}-T_r)}{T_r}$ (cm sec^{-2})	Δx (cm)	$\bar{u}\,\Delta\bar{u}/\Delta x$ (cm sec^{-2})	$\tilde{u}\,\Delta\tilde{u}/\Delta x$ (cm sec^{-2})
19·2	2·7	2·8	19·0 − 7·0	4·3; 3·9	4·6
39·8	1·3	2·3	27·6 − 19·0	2·7; 2·4	3·1
60·5	1·4	2·4	43·2 − 27·6	3·3; 2·7	3·6
82·2	1·7	3·5	61·6 − 43·2	−0·6; −1·0	−0·4
			82·4 − 61·6	3·0; 2·3	3·6

TABLE 2

temperature can hardly be responsible for both the through-flow and for the growth of the boundary layer. I am inclined to favour this view, but some people with whom I have discussed the point are not so sure; they feel that, since the argument is not rigorous, the very fact that it leads to a surprising conclusion suggests that T_0 should be taken as the room temperature. Perhaps, the truth lies somewhere between and part of the elevated temperature makes a contribution to the accelerating buoyancy force.

Table 2 therefore shows, for comparison with $\bar{u}\Delta\bar{u}/\Delta x$, values of both $g\cos\alpha\,(\bar{T}-T_a)/T_a$ and $g\cos\alpha(\bar{T}-T_r)/T_r$. A value for the former can, of course, be calculated if the similarity to Townsend's horizontal plate temperature profile is assumed; with $y_{\max}/y_0$ taken as 15 (on the evidence of figure 9) $g\cos\alpha(2{\cdot}37\theta_0 y_0)/y_{\max} = 1{\cdot}25\,\text{cm}\,\text{sec}^{-2}$. That all the tabulated values exceed this reflects the fact that, round this value of y, there is some system in the departure from Townsend's profile, a point which can be seen in figure 4.† The values of $\bar{u}\Delta\bar{u}/\Delta x$ at $y_{\max}$ in table 2 are calculated from consecutive pairs of profiles, the first figure given in each case is based on an average estimate of $\tilde{u}-\bar{u}$ (from the appendix) and the second on an extreme value to show how much the estimation

† This may or may not be significant, but the large scatter in the lowest x temperature profile (for which the point is most marked) goes against a definite conclusion. This scatter is due to even 10 min ($> 10^4\,y_0/u_0$) being not really a long enough averaging period when the turbulence consists almost entirely of large-scale motions.

might affect the issue. Values of $\tilde{u}\,\Delta\tilde{u}/\Delta x$ are also given as this is the one quantity that can be obtained without some guesswork.

Whether or not the evidence of table 2 is regarded as supporting the view that $\overline{u}\,\partial\overline{u}/\partial x$ exceeds $g\cos\alpha(\overline{T}-T_0)/T_0$, the occurrence of the velocity maximum so far from the wall certainly implies inequality of eddy conductivity and eddy viscosity (for $n = 1$, the former is proportional to y^2). A very rough calculation of the Reynolds stress term in (2) on the assumption of equality gives 15 cm sec^{-2}.

The implications of these observations on the way in which momentum is transported by the turbulence will be taken up again in § 13.

First, however, the behaviour further up the plate must be considered. Figure 9*c* shows that the trend is reversed here; there is now deceleration in the region of the velocity maximum. The profile at $x = 117{\cdot}0$ cm is, for instance, much closer to that at $x = 61{\cdot}6$ cm than to the two intermediate ones (at least for $y/y_0 < 50$).

It is essentially impossible to be sure that this is not an upper end effect, that it would still occur at the same x on a much longer plate. But the data shown in figure 2 provide a strong reason for rejecting this explanation. They demonstrate that the phenomenon starts at a position where the removal of the upper screen (figure 1*a*–3) produced negligible change in the profile.

A much more satisfactory explanation—and one that implies that the deceleration is a genuine feature of the ideal case—is that the small-scale turbulence is now intense enough to have an appreciable 'smoothing' effect on the mean velocity.

It has been remarked that the through-flow varied with the general circulation of air in the room. Conclusions reached by comparing profiles are thus open to the objection that there may have been changes between consecutive runs. However, I think the changes in the through-flow took place over periods like that between the separate surveys (a few months) rather than during the course of a single survey (lasting a week to a fortnight). Support for this is provided by the results of the subsidiary survey (figure 10), which also show the important points described above. It may be noted too that a large maximum velocity tends to be associated with a small velocity at the top of the side walls (presumably as a result of continuity), whereas the reverse would be expected if the observations were to be explained by changes in the through-flow.

It is interesting that the increase in $u_{\max}$ over the lower part of the plate (as judged by profiles at $x \sim 30$ cm and $x \sim 80$ cm) is greater in figure 10 than in figure 9.† This may be associated with the lower through-flow velocity, a similar value of $\overline{u}\,\partial\overline{u}/\partial x$ giving a higher $\partial\overline{u}/\partial x$.

12. An observation on large eddy structure

In the above discussion of the through-flow, mention was made of an experiment in which smoke was introduced at the level of the top of the side-walls. An incidental observation during this experiment seems to merit mention; it suggests that the turbulence contains another large-scale motion in addition

† There is very little difference in $\tilde{u}-\overline{u}$ for the two fibres.

to the plumes, though it does not provide any details about it. The observations were made around $x = 75$ cm.

Most of the time the smoke moved, in a highly eddying fashion, in a positive x- and negative y-direction at angles between 0 and 45° to the x-axis. Occasionally, sharp motion occurred carrying smoke away from the plate; these were almost certainly the plumes described by Townsend extending to a height greater than that of the side-walls. About equally frequently, sharp motions of a different type were observed; in these the smoke was carried almost perpendicularly towards the plate until y was less than 4 or 5 cm when its x-velocity increased suddenly and it was carried off up the plate.

13. Momentum transfer by the turbulence

The observations of § 11 provide nails for the coffin of eddy viscosity. The work of Bourne (1959) and Bradshaw & Gee (1962) has simultaneously been providing others. The attempted theory mentioned in § 6 was done before this evidence had accumulated; it is probably true that, aside from my own experiments, one would not now have great hopes of such a theory.

The specific process of equating eddy viscosity to eddy conductivity has failed through two distinct causes. One is straightforward and is well demonstrated by the deceleration of the boundary layer towards the upper end of the plate. The small eddies make an appreciable contribution to the momentum transfer although not to the heat transfer. Moreover, because of the time taken for these to develop, this is an effect that varies with x. The mixing by small eddies can be likened to molecular transfer and so it is still conceptually useful to think of the turbulence acting on the mean flow like a viscosity. The complication here is only that this cannot be equated to the eddy conductivity.

The second point is such that the use of eddy viscosity fails physically as well as mathematically; only if it is allowed to be negative in part of the flow can it be retained, and it is then a concept of doubtful value. It is understandable that this effect should show up over the lower part of the plate where the small eddies have not developed much. For the moment, we are proceeding on the assumption that the evidence for the acceleration exceeding the buoyancy around the velocity maximum *is* convincing. Then there must be momentum transfer to the maximum by the turbulence, *against* the gradient. Since the trouble arises from the velocity maximum being in a region of low buoyancy, this transport is clearly outwards from closer to the wall. The large plumes of the turbulence seem a likely mechanism, though their detailed action can only be surmised. $\overline{u'v'}$ must have a particularly large value on the inside of the maximum (where, from an eddy viscosity viewpoint, it should be negative). The plumes could easily produce this since their buoyancy would give them a high u' simultaneously with giving them a high v'. The observed behaviour requires not only this, but also that $\overline{u'v'}$ decreases with increasing y (see equation (2)), and it is not evident why the plumes should behave in a way to bring that about.

If the evidence for the excess acceleration is not considered convincing, there is no need to invoke negative $\partial(\overline{u'v'})/\partial y$. As to the behaviour of $\overline{u'v'}$, there is no way of saying anything, and it remains possible that $-\overline{u'v'}$ does everywhere have the same sign as $\partial\bar{u}/\partial y$. It remains possible, but not likely; a comparison of the development of the boundary layer (figure 9) with the distribution of buoyancy (figure 3) still leaves the impression that the action of the turbulent momentum transport must be such that the buoyancy close to the wall contributes to the acceleration of fluid further out.

This situation is, in any case, close to ones for which other people have suggested that all is not well with the eddy viscosity concept. Bourne (1959) analysed Griffiths & Davis's (1922) data on free convection from a vertical heated plate. Bradshaw & Gee (1962) observed non-coincidence of the zero stress position and the velocity maximum in a wall-jet. It should be added that in neither of these cases is the position entirely settled. It is doubtful whether Griffiths & Davis's results, which were obtained before the need for careful averaging was realized, are accurate enough to permit analysis involving second derivatives. And other wall-jet experiments by Schwarz & Cosart (1961) are not in complete agreement with Bradshaw & Gee's.

A change in sign not only of $\overline{u'v'}$ but also of $\partial(\overline{u'v'})/\partial y$ would represent a more radical departure from eddy viscosity predictions than occurs in the flows considered by Bourne and Bradshaw & Gee. That the case is more extreme than the vertical plate convection is, in any case, indicated by a comparison of the values of $(\bar{T}-T_0)/(T_1-T_0)$ at the velocity maximum. My conclusions stem considerably from the fact that this is very low ($\sim \frac{1}{20}$) whereas in Griffiths & Davis's results it is typically $\frac{1}{3}$.

There are, further, physical reasons why it should be a more extreme case. The flow differs in two respects from flows in which it has proved useful to equate eddy diffusivities—the velocity and temperature profiles are of different shapes and the turbulence is generated by thermal instability and not by shear. The vertical plate convection has the first difference, but not the second. A region of momentum transport against the gradient is a region of positive $\overline{u'v'}\ \partial\bar{u}/\partial y$; there is thus transfer of energy from the turbulence to the mean flow. There is more scope for such a region when the turbulence is gaining energy from buoyancy than when this must be compensated by advection and transport.

Even in an ordinary turbulent boundary layer, there are indications that the mechanics of the large eddies are complex. The space-time correlation measurements of Favre, Gaviglio & Dumas (1957, 1958) suggest that in parts of the boundary layer, the large eddies may make a contribution to the Reynolds stress opposite in sign to the total,† though sufficiently small not to lead to complications like those discussed above. This is not certain; but, if it is so, a turbulent motion that consists predominantly of large eddies may indeed behave in an unfamiliar way.

† In its simplest form, the evidence lies in the fact that the maximum maximorum (Favre *et al.*'s parlance) moves downstream faster than the mean velocity and also outwards. However, a full examination of the data suggests that one has to argue more circuitously than this. The matter is at present under investigation.

14. Reconsideration of the temperature results

The presuppositions of §§8–10 are now seen to be not wholly consistent with the conclusions about the velocity field; in particular the assumption that the heat is transferred away to infinity perpendicular to the plate is invalidated by the action of the through-flow. However, the revision of the general picture thus occasioned need not be too drastic. The range of y over which the heat is transferred in the x-direction is about 30 cm whereas the origin of the plumes controlling the heat transfer is probably less than 0·5 cm from the plate. Thus there is likely little alteration in the conditions of their production and it remains reasonable that the heat transfer should be close to that from a horizontal plate.

Comments made about the mean temperature profile need rather more qualification. There can be seen in figures 3–6, small differences between profiles at different x. Considering the temperature profiles alone, these do not seem sufficiently systematic to be significant, but in the knowledge of the complexity of the velocity field, maybe they are. The same comment perhaps applies to the small variations in C, the heat-transfer coefficient. Certainly, conclusions based on the similarity of the temperature profile to that over a horizontal plate have less force than it seemed on the evidence of §§8–10 alone, for at the position of the outermost points of figure 3*b*, the outward heat transfer will have been reduced by nearly $\frac{1}{2}$. On the other hand, it should be remembered that figure 4 compresses the variation at the largest values of y into a very small part of the graph; hence, over much of the range for which the approximation to the $1/y$-profile seems satisfactory (with the same coefficient as for a horizontal plate), the fraction of the heat that has been already convected away in the x-direction is much smaller than $\frac{1}{2}$.

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Appendix—Remarks on the relationship between $\tilde{u}$ and $\bar{u}$

The particulars of the fibres used were as follows (in the notation of Tritton 1959).

Fibre 9 (used for main survey, figure 9): $l = 3{\cdot}60$ cm, $d = 0{\cdot}00369$ cm, $\nu_n/\phi_n^2 = 6{\cdot}32\,\mathrm{sec}^{-1}$.

Fibre 10 (used for subsidiary survey, figure 10): $l = 1{\cdot}95$ cm, $d = 0{\cdot}00194$ cm, $\nu_n/\phi_n^2 = 12{\cdot}33\,\mathrm{sec}^{-1}$.

Suitable parabolic approximations (see Tritton 1963*a*) to their responses are respectively (with h in cm, u in cm sec^{-1})

$$h = 1{\cdot}68\times 10^{-3}u + 4{\cdot}44\times 10^{-5}u^2; \quad h = 1{\cdot}50\times 10^{-3}u + 2{\cdot}94\times 10^{-5}u^2.$$

Indications of the size of $\overline{u'^2}$ may be obtained in two ways. (i) Townsend (1959*a*) estimates that over a horizontal plate $(\overline{u'^2})^{\frac{1}{2}}$ is about $5{\cdot}5u_0$ during the 'active' periods and $7u_0$, or somewhat more, during the 'quiescent'. For the inclined plate this suggests that the overall $(\overline{u'^2})^{\frac{1}{2}}$ will approach a value of around $7u_0$ (or about 15 cm sec^{-1}) towards the upper end of the plate, at around $x = 100$ cm judging by figure 7. It will be less further down the plate, by perhaps a factor of 5 near the lower edge. (ii) Values of $(\overline{h'^2})^{\frac{1}{2}}/\overline{h}$, very rough but useful for this purpose, may be estimated from the spot readings. Information obtained with fibre 10 is better for this purpose, as there was some stimulation by the turbulence of resonance of fibre 9, indicating that inequality (14) of Tritton (1963*a*) was not fully satisfied.† With particular attention paid to the vicinity of the velocity maximum, the results indicate that $(\overline{h'^2})^{\frac{1}{2}}/\overline{h}$ is, with perhaps 50 % error, 0·10 at $x \sim 6$ cm, 0·15 at $x \sim 30$ cm, 0·20 at $x \sim 80$ cm, and 0·23 at $x \sim 115$ cm. Our knowledge of the structure of the turbulence allows a guess to be made that (in

x (cm)	$\tilde{u}_{max}$ (cm sec^{-1})	Possible range of $\overline{u}_{max}$ (cm sec^{-1})
19·0	26·0	25·7 to 26·0
43·2	29·0	28·5 to 29·0
82·4	31·2	29·7 to 30·7
117·0	29·5	27 to 28·5

TABLE 3

the notation of Tritton (1963*a*)) $a/l > \frac{1}{2}$ near the lower end of the plate and $> \frac{1}{10}$ near the upper end. The considerations of Tritton (1963*a*) (in particular, figure 2 and equation (6)) now indicate that $(\overline{u'^2})^{\frac{1}{2}}$ is probably in the range 1·5–3 cm sec^{-1} close to the lower edge, 3–5 cm sec^{-1} at $x \sim 30$ cm, 6–11 cm sec^{-1} at $x \sim 80$ cm, and 8–15 cm sec^{-1} at $x \sim 115$ cm. These estimates are somewhat lower than those given by (i) but quite consistent with them.

For fibre 9, $\overline{u}$ corresponds to a value of h that is $4{\cdot}44 \times 10^{-5}\overline{u'^2}$ lower than that corresponding to $\tilde{u}$. Hence, the above remarks give at least upper and lower limits to the amounts the velocities plotted in figure 9 exceed the true mean velocities. As examples of this, table 3 gives estimates of the possible range of $\overline{u}_{max}$ for various values of x.

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† This observation is of some interest in its own right as indicating the frequencies present in the turbulence. The first resonances of fibres 9 and 10 were at 22·2 and 43·4 c/s respectively. The amplitudes of the resonant vibrations of fibre 9 were small compared with the lower-frequency motions, so that the considerations of §7 of Tritton (1963*a*) imply that there was only a small proportion of the turbulent energy in frequencies as high as 20 c/s.

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